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Noise-reduced diffusion-limited deposition

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Abstract. To investigate the role of fluctuations in diffusion-limited growth we apply the noise reduction algorithm to the problem of two-dimensional deposition on a substrate consisting of a straight line. We consider the scaling of the height and the width of trees growing on the square lattice. For the sizes studied we find two length scales characterised by different exponents depending on the noise reduction parameter m. Using these exponents the effective fractal dimension D_{eff} of trees is calculated and for $m \gg 1$ we obtain $D_{eff} \approx 1.57$. The density profile in the direction of growth and the scaling properties of the lateral correlation function are also determined.

1. Introduction

Laplacian or diffusion-limited growth processes are of fundamental importance leading to a variety of interesting phenomena such as the development of fractal structures or the formation of diverse patterns (for recent reviews see Meakin (1987a), Kertész (1987) and Vicsek (1987)). The widely studied related computer model is diffusion-limited aggregation (DLA) introduced by Witten and Sander (1981). Despite its simplicity this model shows a number of non-trivial features and it represents a useful basis for understanding a variety of non-equilibrium growth processes leading to complex geometries. Noise reduction (Tang 1985, Szép *et al* 1985, Kertész and Vicsek 1986, Nittmann and Stanley 1986) has been shown to reveal a number of important features of DLA and we expect to obtain relevant information from the application of this method to the problem of diffusion-limited deposition.

In the original DLA algorithm particles coming from 'infinity' are added, one at a time, to a growing cluster or aggregate (starting from a single seed) via random walks. One of the most important questions concerning DLA is the role of fluctuations and anisotropy in the formation of clusters and the related problem of their asymptotic shape. Initially DLA clusters were believed to be statistically self-similar isotropic fractals (Witten and Sander 1981, 1983, Meakin 1983) and the fractal dimension D was found to be independent of microscopic details such as the structure of the underlying lattice (Meakin 1983) or the sticking probability (Witten and Sander 1983). The open fractal-like structure of the clusters may be regarded as being a result of consecutive tip splitting and screening instabilities. However, later careful studies showed that the correlations in the clusters are not isotropic (Meakin and Vicsek 1985, Halsey and Meakin 1985, Kolb 1985) and that the overall shape of the cluster is affected by the lattice on which the clusters are grown (Meakin 1985, Ball and Brady 1985). Turkevich and Scher (1985) gave theoretical arguments for the idea that the DLA

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process is not universal, i.e. the exponent D may depend on the lattice structure. Ball *et al* (1985) showed that anisotropic sticking probabilities lead to needle-shaped clusters with

$$H \sim s^{\nu_\parallel} \tag{1}$$

$$W \sim s^{\nu_\perp} \tag{2}$$

where H and W are the length and width of the needle respectively, s is its mass and $\nu_\parallel (=\frac{2}{3}$ in two dimensions) and $\nu_\perp (=\frac{1}{3})$ are the corresponding exponents.

On lattices of low symmetry very large clusters generated by the DLA process grow into star-like objects with elongated arms in the direction of the lattice axes (Meakin 1986, Meakin *et al* 1987). However, for small sizes the radius of gyration R_g grows with increasing cluster mass s according to

$$R_g \sim s^{\nu} \tag{3}$$

where the exponent $\nu (\nu = 1/D)$ is independent of the lattice and its value is close to $\nu = 0.585$ which was obtained for all attainable sizes in off-lattice simulations (Meakin and Sander 1985). Thus at early stages of the growth the fluctuations suppress the anisotropy and the lattice anisotropy becomes important only for very large sizes leading to clusters with an overall dendritic shape (Meakin *et al* 1987). The actual shape of DLA clusters of a given size on lattices results from an interplay between anisotropy due to the underlying mesh and the fluctuations inherent in the growth algorithm (Kertész and Vicsek 1986).

During the last two years we have learned much about the asymptotic structure of DLA but the final answer is still lacking. One way to study the interplay between anisotropy and fluctuations is to generalise the algorithm in order to control the noise. This can be done by introducing Monte Carlo averaging as was independently proposed for diffusion-limited growth by Tang (1985) and Szép *et al* (1985). The reduction of noise has the effect that the lattice anisotropy shows up for much smaller sizes than in the original DLA and it appears that the asymptotic behaviour sets in at earlier stages of the growth (Kertész and Vicsek 1986, Kertész *et al* 1986b). Using this model Thompson (1987) obtained data supporting the assumption that the asymptotics does not correspond to needles with two different exponents corresponding to (1) and (2) in accord with Nittmann and Stanley's (1986) observation on a related model. This result was later reinforced by Meakin (1987a, b). Ball (1986) used the noise-reduced DLA to illustrate ideas on the asymptotic number of branches in diffusion-limited aggregates.

In most of the above studies the geometry corresponded to aggregates growing out of a single seed. However, from the physical point of view the diffusion-limited growth of deposits on flat substrates is also of considerable importance (Meakin 1983, Rácz and Vicsek 1983, Jullien *et al* 1984). In this case trees or clusters are growing on the substrates and their statistics and scaling behaviour are related to the fractal properties of the aggregates (Rácz and Vicsek 1983, Vicsek 1983, Meakin and Family 1986). The exponent α governing the decay of the density in the growth direction is related to the fractal dimension: $\alpha = d - D$. Kertész *et al* (1986a) applied a noise reduction algorithm to a closely related model on the square lattice. The purpose of the present paper is to give an account of a systematic study of noise-reduced diffusion-limited deposition.

The rest of the paper is organised as follows. In § 2 we define the model and describe the quantities of interest. The results are presented in § 3. Finally, in § 4, a discussion and summary is given.

2. The model

Diffusion-limited deposition in d dimensions is defined in the following way (Meakin 1983, Rácz and Vicsek 1983). Initially the growth sites are positioned on a $(d-1)$-dimensional flat substrate (a line in two dimensions) of linear size L. The growth proceeds as randomly walking particles launched from a distant $(d-1)$-dimensional plane hit the surface of the deposit and stick to it. In practice it is not important to let the walkers start far from the growing object; only the level where the walkers are 'killed' has to be far enough away. The algorithm we used is an appropriate modification of Meakin's (1985) 'semi-lattice' simulation of growing DLA clusters to this different geometry. Throughout this paper we are concerned with growth on the two-dimensional square lattice.

Noise reduction (Tang 1985, Szép *et al* 1985) is introduced into this model in a natural way. Instead of adding the particle to the deposit immediately after it hits a growth site we keep a record of how many times each of the surface sites becomes a termination point for a random walker. After an unoccupied surface site has been contacted m times it is filled and the new unoccupied surface sites are identified. The scores (number of contacts) associated with these sites are set to zero. The scores associated with all of the other surface sites remain at their values before this event. In this form the algorithm is an application of the model used first by Kertész and Vicsek (1986) to study the asymptotics of DLA clusters.

Each realisation of our simulation leads to a forest of trees or clusters (collection of particles connected to the same site of the substrate via nearest neighbours). Our aim was to give a statistical analysis of these objects. For this purpose we examined the number of trees of a given size, the number of trees higher or broader than a certain value, the average length and width of the trees, the density as a function of the distance from the substrate and the lateral correlation function.

3. Results

As was discussed above, noise reduction alters the general appearance of DLA clusters and similar changes can also be observed in the case of deposits. This is demonstrated in figure 1, where a typical simulation of the noise-reduced diffusion-limited deposition process is shown. The structure displayed in this figure is reminiscent of dendritic patterns with stable tips and decreased level of randomness and is similar to the corresponding picture obtained by a related algorithm in Kertész *et al* (1986a). The configuration consists of individual tree-like clusters with almost perfectly straight main stems and the fluctuations are manifested in the random distribution of the positions of trees and branches. As the noise is further reduced the trees become even more needle-like with a relatively smaller width and a decreased sidebranching rate.

The two-dimensional simulations were carried out on a strip of width $L = 4096$ lattice units with periodic boundary conditions. In most cases the growth was stopped at a maximum height $h_{\max} \approx 700$ lattice units in order to satisfy the condition $h_{\max} \ll L$

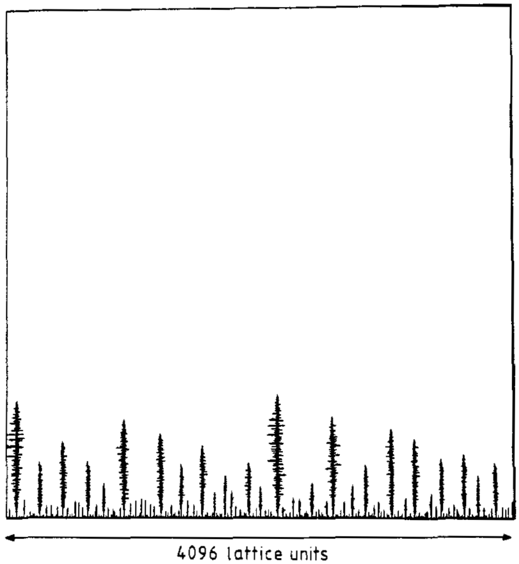

Figure 1. A typical deposit grown with the noise reduction algorithm. This dendritic structure was obtained by using periodic boundary conditions on a strip of width 4096 lattice units and a noise reduction parameter $m = 10$.

which corresponds to the limit in which the effects of the finite strip width can be neglected. In this limit the deposit is quasi-one-dimensional and translationally invariant along the deposition line.

The distribution of particle density is very inhomogeneous in the direction perpendicular to the substrate. This can be studied by calculating the number of particles at a height h, $\rho(h) = \Sigma_x \rho(h, x)$, where $\rho(h, x) = 1$ if the lattice site at (h, x) is filled and is equal to zero otherwise. Figure 2 shows the dependence of the logarithm of the density distribution $\rho(h)$ on log h for the three selected values of the noise reduction

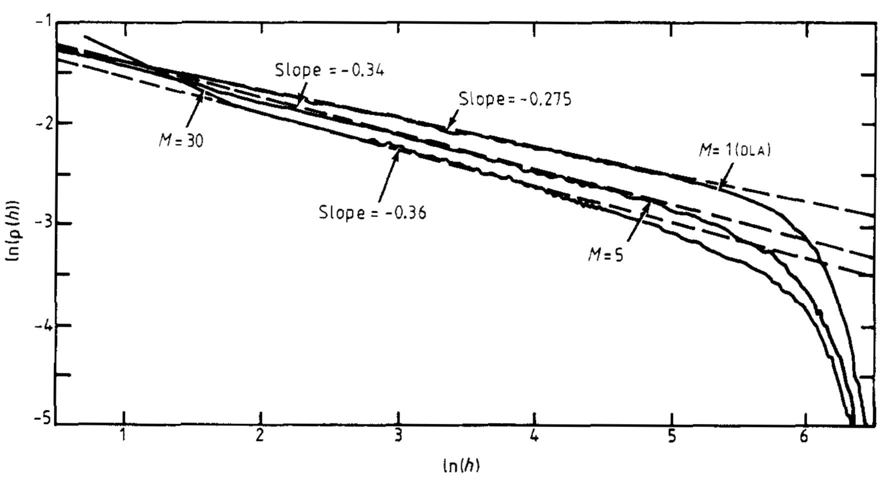

Figure 2. The density $\rho(h)$ as a function of the distance from the line onto which the particles are deposited. A strip width of 4096 lattice units and a deposit height of 700 was used to obtain the results shown in this and the subsequent figures.

parameter m. This figure suggests that, for $h \ll L$, $\rho(h)$ behaves as

$$\rho(h) \sim h^{-\alpha} \tag{4}$$

with a non-universal exponent α depending on m.

In order to describe the statistics of clusters generated for various values of m we studied the following distribution functions which can be defined in the deposition model. First we determined $N_h(l)$ the number of clusters having a height of l lattice units and $N_w(l)$ which is the number of clusters of width l. The results for $m = 10$ are shown in figure 3 which indicates that both quantities scale as a function of l with an exponent close to -1.8. Figure 4 shows the number of trees consisting of s particles, $N(s)$, which was found to decay as $N(s) \sim s^{\tau}$ with $\tau \approx -1.64$. This value is very close to the corresponding exponent for the $m = 1$ case (Meakin 1984).

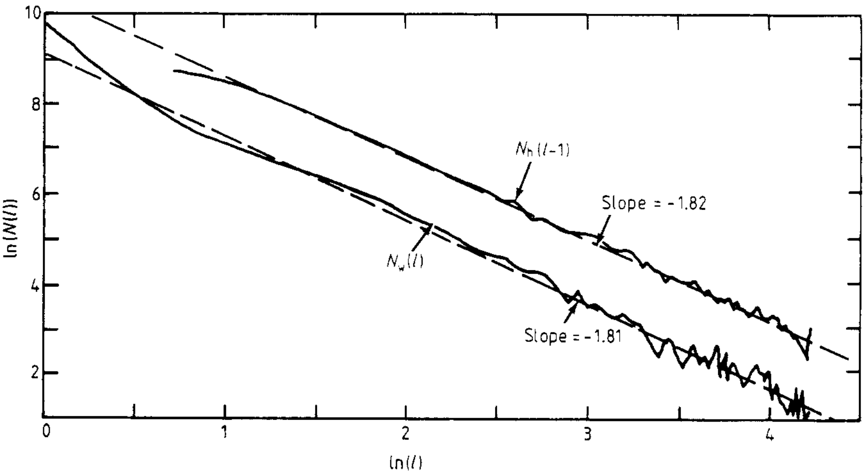

Figure 3. The distribution of cluster heights $N_h(l)$ and cluster widths $N_w(l)$ for $m = 10$, where $N_h(l)$ is the number of trees l lattice units in height and $N_w(l)$ is the number of trees l lattice units in width.

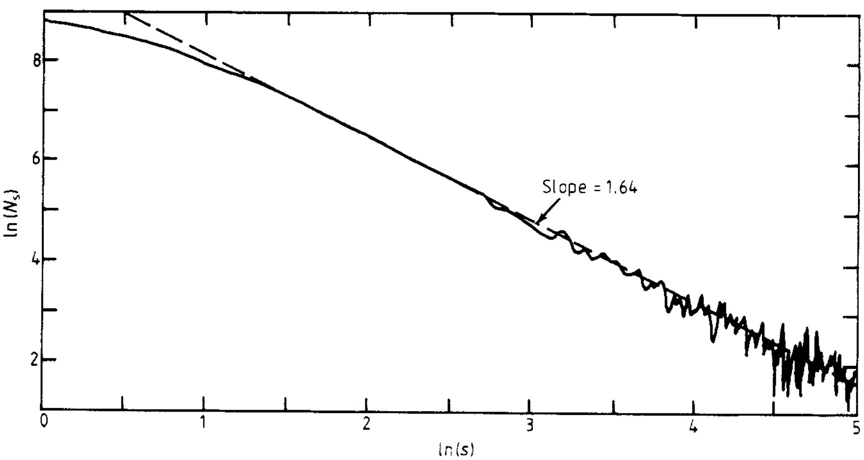

Figure 4. The dependence of the cluster size distribution function $N(s)$ on the number of particles in a cluster for $m = 10$.

The above results suggest that there is a non-trivial scaling between the mass of a cluster and its linear sizes. Therefore we calculated the dependence of mean tree height (H) and width (W) on the number of particles in the tree. A typical plot of these quantities is presented in figure 5. This figure represents clear evidence in favour of the assumption that, for the considered sizes, at least two independent scaling lengths are needed to describe the geometry of clusters in a deposit with noise reduction. The runs with various values of m indicate that, for $s \gg 1$,

$$H \sim s^{\nu_\parallel} \tag{5}$$

and

$$W \sim s^{\nu_\perp} \tag{6}$$

with the effective exponents depending weakly on m. Exponents for several m values are given in table 1. The complex behaviour of the noise reduction model is demonstrated by this table in which the values of the quantities do not depend monotonically on m.

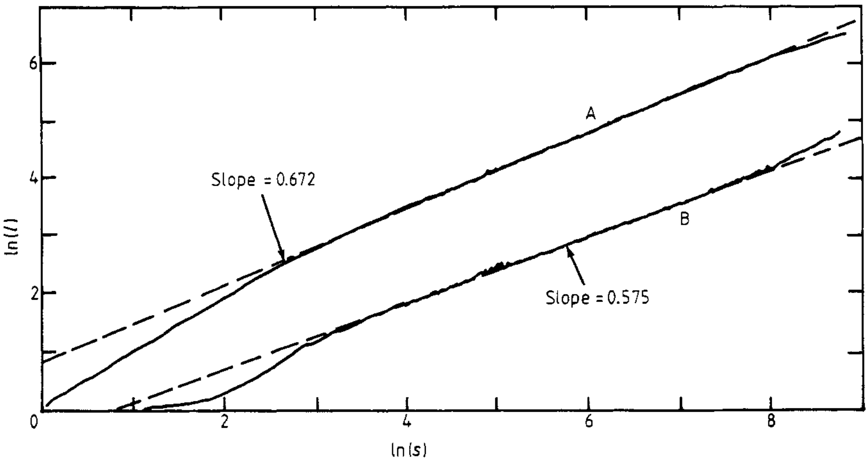

Figure 5. This figure demonstrates the scaling of (A) the mean tree height, H, and (B) the width, W, for $m = 10$ as a function of the number of sites belonging to the trees (s).

Table 1. The exponent $\nu_\parallel$ ($\nu_\perp$) describing the dependence of the length (width) of trees on the number of sites s. The effective fractal dimension D_{eff} calculated using (9) is also indicated.

m	$\nu_\parallel$	$\nu_\perp$	D_{eff}
1	0.64	0.56	1.64
2	0.64	0.56	1.64
3	0.65	0.56	1.63
5	0.67	0.55	1.60
10	0.67	0.57	1.58
20	0.66	0.60	1.57
30	0.65	0.61	1.57

Next we investigated the correlations along the lateral direction x (parallel to the deposition line) using the expression

$$C_h(x) = \frac{1}{L}\sum_{x'} \rho(h, x+x')\rho(h, x') \tag{7}$$

This function is related to the tangential correlation function which was introduced for the usual DLA clusters (Meakin and Vicsek 1985, Kolb 1985) in order to describe the internal anisotropy of diffusion-limited aggregates. The results for the lateral correlation function with $m=1$ and $m=20$ are shown in figures 6(a) and 7(a) respectively.

The $C_h(x)$ curves we plotted exhibit a number of interesting features. For all values of h they have a well pronounced minimum followed by a less apparent maximum. The position of the minima $x_{\min}(h)$ depends on the height at which the correlation function was calculated. To study the possible scaling of $x_{\min}(h)$ we plotted this

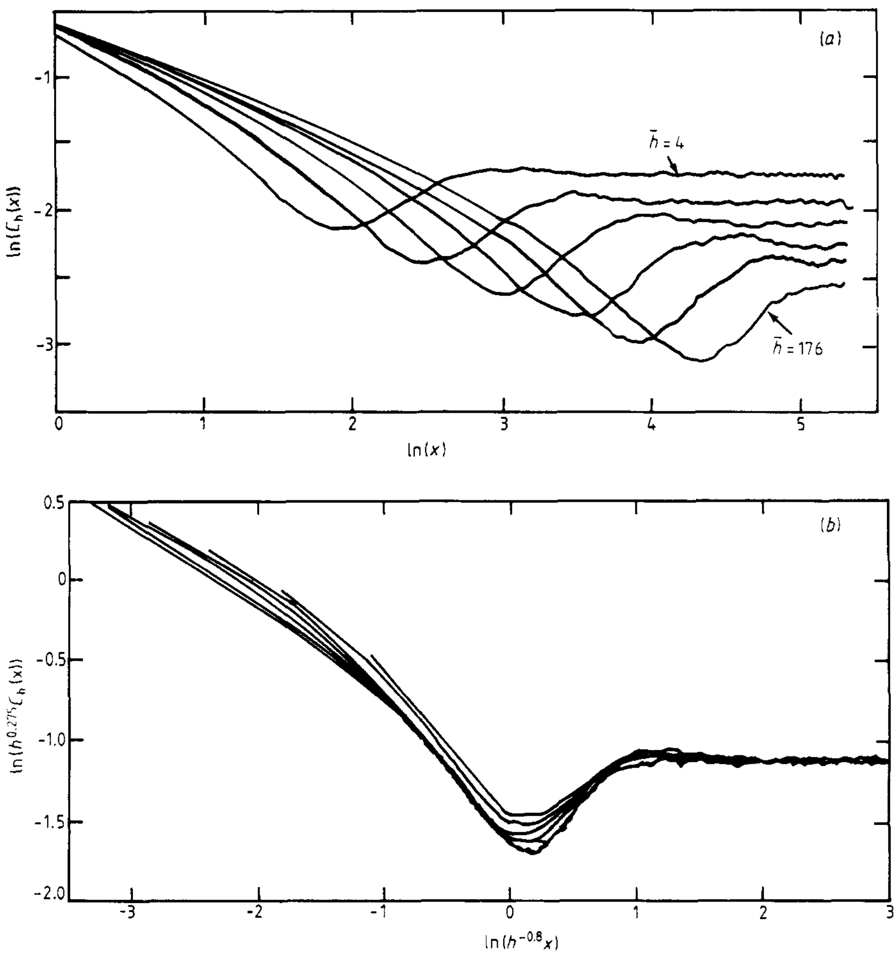

Figure 6. The lateral correlation functions $C_h(x)$ for $m = 1$ (DLA), where h is the height at which the correlations in the x direction (along the deposition line) were determined are displayed on (a). (b) shows the scaling of the correlation function according to the scaling form given in equation (8).

quantity against h on a log-log plot (figure 8) for three selected values of m. It can be seen from this figure that the position of the minima scales with h according to an exponent $0.8 < \delta < 0.9$ if m changes between 1 and 20. It is plausible to assume that $x_{min}(h)$ is proportional to the mean distance of the trees at the height h. Then, e.g., for $m = 20$ the exponent $\delta \approx 0.9$ is consistent with the previously found $\nu_\parallel/\nu_\perp \approx 0.89$ which means that the width and the separation of the trees scale as a function of h approximately the same way.

We have also attempted to scale the correlation function $C_h(x)$ measured at different heights (h) onto a common curve. Figure 6(b) shows that for the case $m = 1$ (diffusion-limited deposition) the correlation function can be described quite well in terms of the scaling form

$$C_h(x) \sim h^{-\alpha} f(x/h^{\delta}) \tag{8}$$

where the exponents α and δ have values of 0.275 and 0.8, respectively. Figure 7(b)

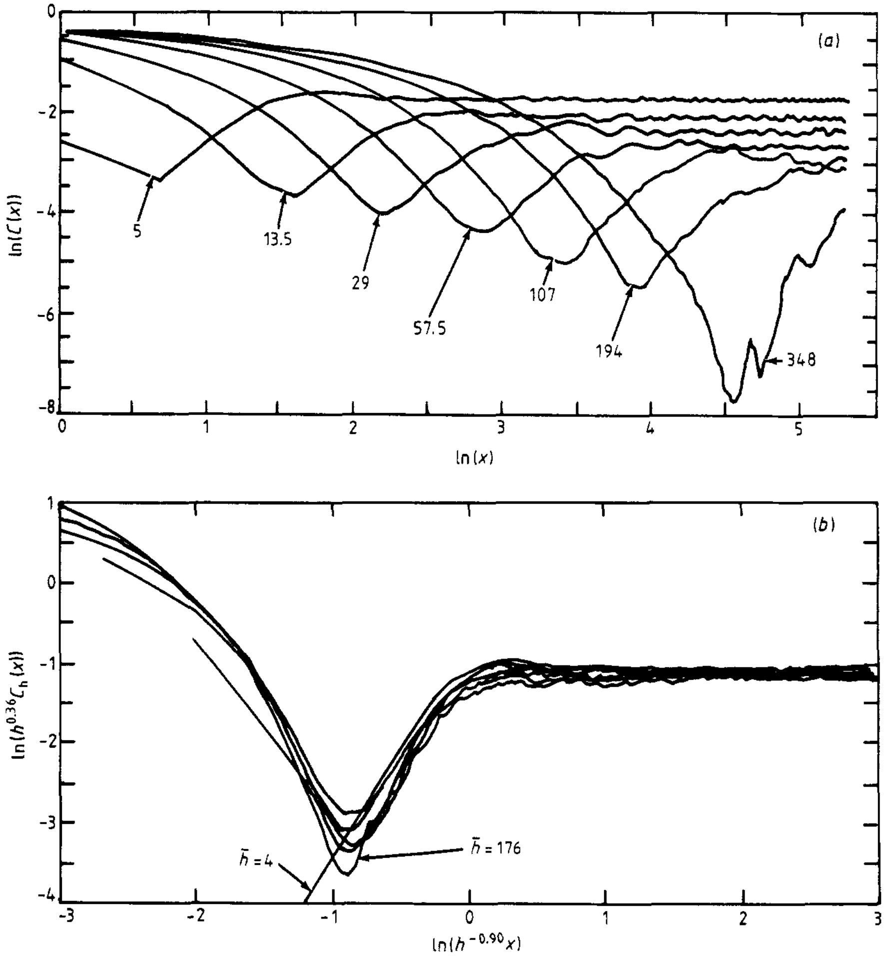

Figure 7. (a) shows the correlation functions $C_h(x)$ obtained at eight different values of $\bar{h}$ averaged over a small range for the case $m = 20$. (b) shows similar correlation functions for a noise reduction parameter $m = 30$ which has been scaled using the scaling form of equation (8).

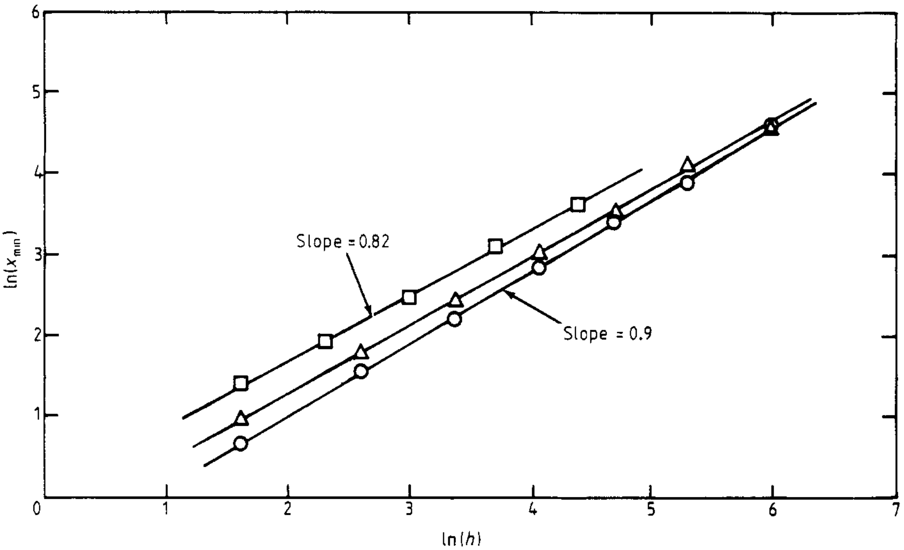

Figure 8. Scaling of the minima $x_{\min}(h)$ of the lateral correlation function with the height h. The straight line fitted to the data indicates that for various m the minima diverge with h according to an exponent $\approx$0.8–0.9. □, $m = 1$; △, $m = 5$; ○, $m = 20$.

shows a similar plot for the case $m = 30$. In this case we find that $\alpha \simeq 0.39$ and $\delta \simeq 0.9$ give the best data collapse. These results indicate that the exponents α and δ are not universal and suggest that in the limit $m \to \infty$ (and for $s \to \infty$) the exponent δ may approach the value of 1.0. If it is so, then only one exponent (α) may be needed in this asymptotic limit. This may be related to the observation that for large noise reduction parameters (m) the exponents $\nu_{\parallel}$ and $\nu_{\perp}$ appear to converge (Nittmann and Stanley 1986, Thompson 1987, Meakin 1987a, b).

Finally, the behaviour of $C_h(x)$ is non-trivial for $x \ll h$. In the case of $m = 1$ the slope of the curve seems to approach the limiting value $\alpha_{\min} \approx 0.42$ which indicates that the decay of correlations in the lateral direction is faster than in the direction parallel to the growth. The situation is less clear for $m > 1$ where the correlation function probably has a zero slope for $x \ll h$. This is an interesting result which has its origin in the dendritic structure of the trees. There is either a branch growing out from the main stem horizontally at a given height (and resulting in a constant local density) or no branch at all (no contribution to the correlation function).

4. Discussion

Our results show that noise reduction applied to diffusion-limited deposition has a dramatic effect on the shape of the clusters and it also changes their scaling properties. Although the trees grown on the substrate and the branches of a noise-reduced DLA cluster are visually similar, there are unexpected differences in the scaling powers.

It is interesting to note that in spite of the well pronounced algebraic dependence of $\rho(h)$ on h the exponents describing this decay do not seem to be trivially related to the decay of density in the corresponding noise-reduced DLA clusters grown from a single seed. For example, in a deposit with $m = 20$, $\rho(h) \sim h^{-0.36}$, while in the radial case, $\rho(R) \sim R^{-0.45}$ (Meakin 1987a, b, Thompson 1987) where R is the distance from the seed particle.

We have demonstrated that the trees are very anisotropic. Thus their width and length scale with different exponents. This seems to be different from the situation with the single-particle seed geometry where recent studies (Thompson 1987, Meakin 1987a, b) indicate a finally isotropic scaling. However, some of our results (from the correlation function $C_h(x)$) do suggest that the scaling might be isotropic in the asymptotic limit $m \to \infty$, $s \to \infty$. Since the two lengths (the height and the width) of the trees scale differently, the clusters are not self-similar fractals and one needs a modified definition for the effective fractal dimension for such objects. Using the expression introduced by Nadal *et al* (1984) for the case of directed lattice animals and DLA clusters

$$D_{\text{eff}} = 1 + \frac{1 - \nu_{\|}}{\nu_{\perp}} \tag{9}$$

where the ν values corresponding to the different noise reduction parameters have to be taken from table 1. The effective fractal dimension decreases as a function of increasing $m = 1$ and seems to converge to the value 1.57. The data of table 1 should be compared with the asymptotic fractal dimension ~ 1.55 of noise-reduced DLA clusters on the square lattice. We find that the noise-reduced deposits are somewhat less compact than ordinary diffusion-limited aggregates. Furthermore, they do not approach, for the considered sizes, the ideal needle limit with $\nu_{\|} = \frac{2}{3}$ and $\nu_{\perp} = \frac{1}{3}$ (Turkevich and Scher 1985, Ball *et al* 1985). Meakin and Family (1986) obtained, for DLA $(m = 1)$ in the limit $L \ll h$, different scaling in the parallel and perpendicular directions. On the basis of our simulations carried out for $L \gg h$ we conclude that—at least for the sizes we studied here—the trees grown by the noise-reduced diffusion-limited deposition are self-affine fractals (Mandelbrot 1986).

Acknowledgments

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References

Ball R C 1986 *Physica* **140A** 62
Ball R C and Brady R M 1985 *J. Phys. A: Math. Gen.* **18** L809
Ball R C, Brady R M, Rossi G and Thompson B R 1985 *Phys. Rev. Lett.* **55** 1406
Halsey T C and Meakin P 1985 *Phys. Rev.* A **32** 2547
Jullien R, Kolb M and Botet R 1984 *J. Physique* **45** 395
Kertész J 1987 *Phil. Mag.* in press
Kertész J, Szép J and Cserti J 1986a *Growth and Form: Fractal and Nonfractal Patterns in Physics* ed H E Stanley and N Ostrowsky (Dordrecht: Nijhoff) p 249
Kertész J and Vicsek T 1986 *J. Phys. A: Math. Gen.* **19** L257
Kertész J, Vicsek T and Meakin P 1986b *Phys. Rev. Lett.* **57** 3303
Kolb M 1985 *J. Physique Lett.* **6** 631
Mandelbrot B 1986 *Fractals in Physics* ed L Pietronero and E Tosatti (Amsterdam: North-Holland) p 3
Meakin P 1983 *Phys. Rev.* A **27** 2616
—— 1984 *Phys. Rev.* B **30** 4207

—— 1985 *J. Phys. A: Math. Gen.* **18** L661
—— 1986 *Fragmentation, Form and Flow in Fractured Media* (*Annals Israel Physical Society* **8**) ed R Englmann and Z Jaeger (Bristol: Adam Hilger) p 556
—— 1987a *Phase Transitions and Critical Phenomena* vol 12, ed C Domb and J L Lebowitz (New York: Academic) to be published
—— 1987b *Phys. Rev.* A **36** 332
Meakin P, Ball R C, Ramanlal P and Sander L M 1987 *Phys. Rev.* A **35** 5233
Meakin P and Family F 1986 *Phys. Rev.* A **34** 2558
Meakin P and Sander L M 1985 *Phys. Rev. Lett.* **54** 2053
Meakin P and Vicsek T 1985 *Phys. Rev.* A **32** 685
Nadal J P, Derrida B and Vannimenus J 1984 *Phys. Rev.* B **30** 376
Nittmann J and Stanley H E 1986 *Nature* **321** 663
Rácz Z and Vicsek T 1983 *Phys. Rev. Lett.* **51** 2382
Szép J, Cserti J and Kertész J 1985 *J. Phys. A: Math. Gen.* **18** L413
Tang C 1985 *Phys. Rev.* A **31** 1977
Thompson B R 1987 *Preprint*
Turkevich L A and Scher H 1985 *Phys. Rev. Lett.* **55** 1026
Vicsek T 1983 *J. Phys. A: Math. Gen.* **16** L647
—— 1987 *Phys. Scr.* **19** 334
Witten and Sander 1981 *Phys. Rev. Lett.* **47** 1400
—— 1983 *Phys. Rev.* B **27** 5686